from the Exhaust Plume of an Aluminized Composite Propellant Rocket," *Journal of Spacecraft and Rockets*, Vol. 1, No. 5, Sept.-Oct. 1964, pp. 501–507.

[3] Edwards, D. K., "Review of Thermal Radiation from TRW INTELSAT III Aluminized Solid Rocket Plume," Rept. 68-3346.1-34, May 13, 1968, TRW Systems Group, Redondo Beach, Calif.

[4] Hottel, H. C. and Sarofim, A. F., *Radiative Transfer*, McGraw-Hill, New York, 1967, p. 381.

[5] Plass, G. N., "Temperature Dependence of Mie Scattering and Absorption Cross Section for Aluminum Oxide," *Applied Optics*, Vol. 4, 1965, pp. 1616–1619.

[6] Chandrasekhar, S., *Radiative Transfer*, Dover Publications, New York, 1960, Chap. V.

[7] Sparrow, E. M. and Cess, R. D., *Radiation Heat Transfer*, Brooks/Cole, Belmont, Calif., 1966, pp. 129–134.

Reply by Author to D. K. Edwards

Richard P. Bobco*
Hughes Aircraft Company, El Segundo, Calif.

The author gratefully acknowledges Dr. Edwards' correction of the shape factor curves in Ref. 1 and the comments on the utility of approximating a metal oxide laden exhaust plume as a conical surface. His enlargement of the concept to introduce an "effective shape factor" should prove useful to thermal designers who must protect spacecraft surfaces from rocket plume radiation.

The analysis of Ref. 1 represented a first, and apparently unsteady, step in attempting to reduce an extremely difficult problem to a level which could be treated by hardware oriented thermal designers. Subsequent studies of the "metal oxide plume problem" are reported in Ref. 2–4, in which the integro-differential equation of radiative transfer is solved by the diffusion-iteration approximation. These studies together with a vastly different problem reported in Ref. 5 support the following conclusions: 1) the emittance from an isothermal, isotropically scattering dispersion is not diffuse (Ref. 2), 2) directional emittance, uniform over a "surface," is analytically equivalent to a surface of nonuniform radiostiy when computing the radiant flux at a remote location (Ref. 5, 2–4), and 3) the value $\omega = 1$ is the best approximation for an isotropically scattering conical plume with an albedo approaching unity and an exit optical-scattering depth of about 3 (Ref. 4).

The author agrees that the reference radiosity J_0 should be a measured quantity rather than computed. However, if J_0 is measured at the exit plane of a rocket nozzle (a common location), the assumption $\omega = 1$ is conservative insofar as the axial decay is more rapid. The exit-plane radiosity is enhanced by radiation from heteropolar gas products that cool rapidly downstream of the exit plane, and the "searchlight" effect of the combustion chamber and nozzle enclosure irradiating particles immediately downstream of the exit plane. Both of these effects become less pronounced several exit diameters downstream of the exit plane. A third influence, that of anisotropic scattering, on the magnitude of ω cannot be assessed properly at this time. The results of an exploratory study of the searchlight effect together with anisotropic scattering are reported by Stockham and Love.[6] However, their cylindrical geometry with constant density is a model of the plume in the vicinity of the exit plane only and does not provide any insight to the magnitude of ω.

Dr. Edwards' product $Q_a\tau$ is an apparent emittance at the exit plane. In Ref. 4 it is identified as the normal emissivity (emittance) at the cone surface. An order of magnitude uncertainty in the imaginary part of the refraction index for fused Al_2O_3 indicates that $0.02 \leq Q_a\tau \leq 0.20$ where $\tau = 3$. This observation, together with the uncertain influence of gaseous radiation and searchlight effect provides additional support for obtaining J_0 from measurements rather than computations.

Finally, the expression for directional emittance, $\epsilon/\epsilon_{normal} = 0.50\ (1 + \cos\theta)$, appears to be a reasonable fit for Chandrasekhar's H-function for semi-infinite plane dispersions, but the emittance from a conical geometry has a more complicated directional character. Emittance curves for a conical geometry are presented in Ref. 4 and these show both polar and azimuthal dependence on angle. In the plane containing the conical axis, the emittance resembles $\epsilon/\epsilon_{normal} \approx 1 + \sin\theta$ while in a plane normal to the axis $\epsilon/\epsilon_{normal} \approx \cos\theta$. However, in view of 1) the equivalence between nondiffuse local emittance and a nonuniform radiosity distribution, and 2) the rapid axial decay of gaseous radiation and the searchlight effect, it appears that Edwards' Figs. 8 and 9 are most useful for thermal design. It should be observed that the combination $\omega = 1$ and $\epsilon = 0.50\ (1 + \cos\theta)$ is roughly equivalent to the choice $\omega = 2$.

Received February 11, 1969.
* Senior Staff Engineer. Associate Fellow AIAA.

References

[1] Bobco, R. P., "Radiation from Conical Surfaces with Nonuniform Radiosity," *AIAA Journal*, Vol. 4, No. 3, March 1966, pp. 544–546.

[2] Edwards, R. H. and Bobco, R. P., "Radiant Heat Transfer from Isothermal Dispersions with Isotropic Scattering," Rept. SSD 50134R, ATL No. 3, Dec. 1965, Hughes Aircraft Co.; abridged version published in *Journal of Heat Transfer, ASME Transactions*, Vol. 89, Ser. C., No. 4, Nov. 1967, pp. 300–308.

[3] Bobco, R. P., "Directional Emissivities from a Two-Dimensional, Absorbing-Scattering Medium: The Semi-Infinite Slab," *Journal of Heat Transfer, ASME Transactions*, Vol. 89, Ser. C, No. 4, Nov. 1967, pp. 313–320.

[4] Bobco, R. P. and Edwards, R. H., "Radiation from an Absorbing Scattering Conical Dispersion with Non-Uniform Density," Rept. SSD 60571R, ATRR No. 21, Dec. 1966, Hughes Aircraft Co.; Rept. abridged version in Paper 68-HT-29, Aug. 1968, American Society of Mechanical Engineers.

[5] Bobco, R. P. and Stensgaard, C. P., Jr., "Solar Incidence Factors for Thermal Analysis of Imperfectly Collimated Radiation in Solar Simulators," *Journal of Spacecraft and Rockets*, Vol. 4, No. 2, Feb. 1967, pp. 236–242.

[6] Stockham, L. W. and Love, T. J., "Monte Carlo Solution of Radiative Heat Transfer from a Finite Cylindrical Cloud of Particles," Paper 68-64, Jan. 1968, AIAA.

Comments on "A Reattachment Criterion for Turbulent Supersonic Separated Flows"

R. H. Page*
Rutgers University, New Brunswick, N. J.

Batham[1] has shown that his reattachment criterion correlates certain experimental data with an empirical constant K. This correlation should be interpreted as one possible correlation in a region of possible solutions. It has been shown[2] that the turbulent pressure coefficient at reattachment depends upon both Reynolds number (initial boundary layer) and Mach number for free reattachments.

Received February 17, 1969.
* Professor and Chairman, Department of Mechanical & Aerospace Engineering. Associate Fellow AIAA.

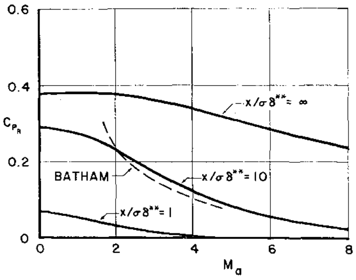

Fig. 1 Reattachment pressure coefficient.

Figure 1 shows values of the reattachment pressure coefficient for three values of $X/\sigma\delta^{**}$. (X is the length of the constant pressure mixing region before reattachment, σ is the customary similarity parameter, and δ^{**} is the boundary-layer momentum thickness at separation.) The highest values of C_{PR} correspond to reattachment of fully developed free shear layers which are independent of initial boundary-layer effects (similar solutions). Intermediate values of C_{PR} are for the developing (or preasymptotic) shear layers. The theoretical curves shown in Fig. 1 are for free isoenergetic reattachments in air ($k = 1.4$) calculated by available methods.[2]

Batham's recent correlation is plotted in Fig. 1. It is apparent that his correlation is for experimental cases in which the shear layer is not yet fully developed. Also one notes that Batham's correlation is for the reattachment point pressure only. Once the reattachment point pressure is known, this must be related to other flow conditions in order to determine a unique solution. An example of such a complete reattachment criterion has been given.[2]

References

[1] Batham, J. P., "A Reattachment Criterion for Turbulent Supersonic Separated Flows," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 154–156.

[2] Page, R. H., Kessler, T. J., and Hill, W. G., "Reattachment of Two-Dimensional Supersonic Turbulent Flows," Paper 67-FE-20, 1967, American Society of Mechanical Engineers.

Reply by Author to R. H. Page

J. P. Batham*

University of Oxford, Oxford, England

PROFESSOR Page presents a correlation of reattachment pressure coefficient in terms of the similarity parameter for similar and nonsimilar flows. However, this parameter is only applicable to flows where the constant pressure shear layer can be represented by a similar (or asymptotic) velocity profile.

The criterion proposed[1] is applicable for large values of X/δ^{**}, where an asymptotic error function velocity profile can be used for the shear layer upstream of reattachment. The experiments of Sirieix et al.[2] show that under these conditions a similar reattachment pressure distribution is obtained.

It is apparent that a large discrepancy exists between the proposed criterion and the correlation of Page et al.[3] based on experiments on rearward facing steps. The correlation of Page et al. gives a relation between the velocity ratio on the discriminating streamline and the angle turned through at reattachment as a fraction of the total turned through at recompression. This discrepancy could be due to the use of asymptotic velocity profiles in calculating the velocity ratio on the discriminating streamline for small step heights where the shear layer may not have been similar, and the use of the turning angle downstream of reattachment in the correlation. The experiments of Sirieix et al.[2] and Roshko and Thomke[4] show that conditions downstream of reattachment may be varied independently of the pressure distribution up to reattachment.

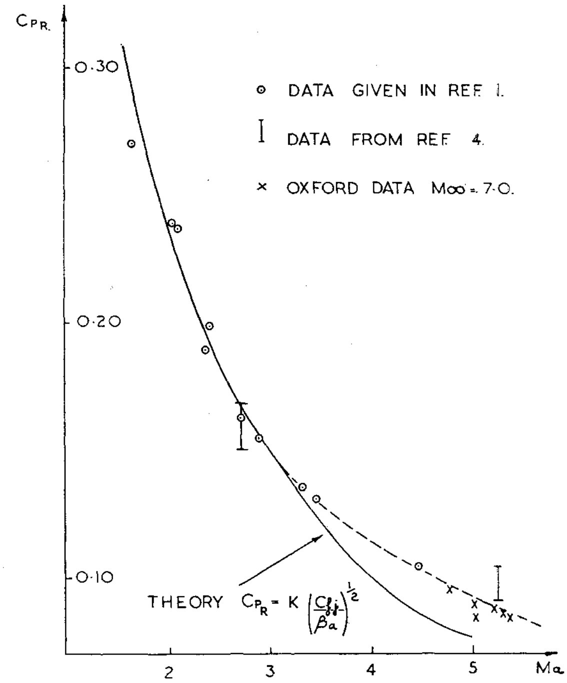

Fig. 1 Reattachment pressure coefficient.

The proposed criterion appears to give a good correlation of data obtained in a large number of facilities. Included in Fig. 1 are two points taken from the data of Roshko and Thomke,[4] where the reattachment point could easily be determined, and some recently obtained data for compression corner flows obtained in the Oxford University Hypersonic Gun Tunnel where the reattachment point was located by means of a surface Pitot. It would not appear to be possible at present to correlate flows with small values of X/δ^{**} owing to the difficulty in evaluating the shear-layer velocity profile.

A solution to the over-all flowfield can be obtained by combining the reattachment criterion with the methods proposed by Childs et al.[5] or Roberts[6] as shown in Ref. 7.

Received April 7, 1969.

* Graduate Student, Department of Engineering Science.

References

[1] Batham, J. P., "A Reattachment Criterion for Turbulent Supersonic Separated Flows," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 154–156.

[2] Sirieix, M., Mirande, J., and Delery, J., "Experiences Fondamentales sur le Recollement Turbulent d'un Jet Supersonique," *AGARD Conference Proceedings No. 4*, 1966, pp. 353–391.

[3] Page, R. H., Kessler, T. J., and Hill, W. G., "Reattachment of Two-dimensional Supersonic Turbulent Flows," American Society of Mechanical Engineers Paper 67-FE-20, Chicago, 1967.

[4] Roshko, A. and Thomke, G. J., "Observations of Turbulent Reattachment behind an Axisymmetric Downstream-Facing Step in Supersonic Flow," *AIAA Journal*, Vol. 4, No. 6, June 1966, pp. 975–980.

[5] Childs, M. E., Paynter, G. C., and Redeker, E., "The Prediction of Separation and Reattachment Flow Characteristics for Two-dimensional Supersonic and Hypersonic Turbulent Boundary Layers," *AGARD Conference Proceedings No. 4*, 1966, pp. 325–352.